CONTRIBUTIONS TO UNCERTAINTY OF MEASUREMENT IN MASS METROLOGY

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1. PURPOSE AND SCOPE

The purpose of this document is specifically to consider contributors to uncertainty of measurements during the process of calibration of weights and weighing instruments.

This document is applicable to Southern African Development Community Accreditation System (SADCAS) accredited Laboratories.

2. DEFINITIONS

Definitions in the following documents apply:

- OIML International Recommendation R111-1: 2004 (E) - Weights of Classes E1, E2, F1, F2, M1, M2, M3;
- SADCAS TR 12 - Estimation of the uncertainty of measurement by calibration laboratories and specification of calibration and measurement capability on schedules of accreditation;
- SADCAS TR 16 Criteria for laboratory accreditation in mass metrology; and
- Guide to the Expression of Uncertainty in Measurement (GUM).

3. FACTORS THAT WOULD AFFECT THE QUALITY OF MEASUREMENT RESULTS

3.1 Environmental Conditions

Calibration of weights should be performed in a laboratory with stable ambient conditions to avoid effect of change in temperature and relative humidity on weighing results. Refer to Table 1 in SADCAS TR 16.

Calibration of weighing instruments is performed at the site where the instruments are used, and environmental conditions may not be controlled.

Where applicable the laboratory shall maintain appropriate records to demonstrate and confirm the humidity, temperature, temperature gradients and ambient pressure within the laboratory.

3.2 Weighing Instruments Used for Calibration of Weights

Metrological characteristics of the weighing instruments, such as resolution, linearity, repeatability and eccentricity should be known.

Each weighing instrument is subject to initial qualification and to periodical requalification.
The metrological performances of the weighing instruments are monitored.

3.3. Reference Weights

Reference weights shall be of higher OIML accuracy class than the weights under calibration. Refer to OIML R111-1.

When the weights are adjusted, the calibration values before and after adjustment are recorded.

3.4 Weighing Process

Accepted procedures for weighing cycles should be considered. See single and double substitution methods described in OIML R111-1.

4. UNCERTAINTY BUDGET FOR CALIBRATION OF WEIGHTS

There are several factors that contribute to measurement uncertainty during calibration of weights. According to the GUM, evaluation of these uncertainties depends on whether they are Type A (based on statistical analysis) or Type B (based on other knowledge). The following are the major contributors that need to appear in the uncertainty budget:

4.1 Repeatability (Standard Uncertainty of the Weighing Process), U_w (Type A)

$$u_{\mathrm{w}}\left(\overline{\Delta m_{\mathrm{c}}}\right)=\frac{s\left(\Delta m_{\mathrm{c}i}\right)}{\sqrt{n}}$$

Where:

4.1.1 for weights of classes F_2, M_1, M_2 and M_3

$$s\left(\Delta m_{\mathrm{c}}\right)=\frac{\max\left(\Delta m_{\mathrm{c}i}\right)-\min\left(\Delta m_{\mathrm{c}i}\right)}{2\times\sqrt{3}}$$

4.1.2 for weights of classes E_1,E_2 and F_1

$$s^2\left(\Delta m_{\mathrm{c}}\right)=\frac{1}{n-1}\sum_{i=1}^{n}\left(\Delta m_{\mathrm{c}i}-\overline{\Delta m_{\mathrm{c}}}\right)^2$$

With *n-1* degrees of freedom

4.2 Uncertainty of reference weight, U(m_{cr}) (Type B)

$$u(m_{\mathrm{cr}})=\sqrt{\left(\frac{U}{k}\right)^2+u_{\mathrm{inst}}^2(m_{\mathrm{cr}})}$$

Note: U(m_{cr}) can simply be found by dividing the uncertainty on the calibration certificate by coverage factor k. K=2 at 95% confidence level.

Uncertainty of instability (drift) of the reference weight, $U_{inst}(m_{cr})$ can be estimated using the calibration history.

4.3 Uncertainty Due to Display Resolution, U_{ba} (Type B)

$$u_{\mathrm{d}}=\left(\frac{d/2}{\sqrt{3}}\right)\times\sqrt{2}$$

Where d = scale interval of the digital balance

4.4 Uncertainty of Air Buoyancy Correction, U_b (Type B)

$$u_{\mathrm{b}}^2=\left[m_{\mathrm{cr}}\frac{(\rho_{\mathrm{r}}-\rho_{\mathrm{t}})}{\rho_{\mathrm{r}}\rho_{\mathrm{t}}}u(\rho_{\mathrm{a}})\right]^2+\left[m_{\mathrm{cr}}(\rho_{\mathrm{a}}-\rho_0)\right]^2\frac{u^2(\rho_{\mathrm{t}})}{\rho_{\mathrm{t}}^4}+m_{\mathrm{cr}}^2(\rho_{\mathrm{a}}-\rho_0)\left[(\rho_{\mathrm{a}}-\rho_0)-2(\rho_{\mathrm{al}}-\rho_0)\right]\frac{u^2(\rho_{\mathrm{r}})}{\rho_{\mathrm{r}}^4}$$

Where:

M_{cr} = conventional mass of reference weight
ρ_r = reference weight density
ρ_t = test weight density
$U(\rho_a)$ = uncertainty of air density (Note the formula on page 68 in OIML R111)
ρ_a = air density
ρ_o = air density at sea level = 1.2 kg m^{-3}
$U(\rho_t)$ = uncertainty of density of test weight
ρ_{al} = air density during previous calibration
$U(\rho_r)$ = uncertainty of density of reference weight

4.5 Uncertainty Due to the Drift of Reference Weights.

The drift uncertainty may be estimated from its history and based on at least two successive calibrations. In the absence of history an estimate may be made to the order of magnitude of the calibration uncertainty.

4.6. Combined Standard Uncertainty, $U_c(m_{ct})$

$$u_c\left(m_{ct}\right)=\sqrt{u_w^2\left(\overline{\Delta m_c}\right)+u^2\left(m_{cr}\right)+u_b^2+u_{ba}^2}$$

4.7 Expanded Uncertainty, $U(m_{ct})$

$$U(m_{ct}) = k\,u_c(m_{ct})$$

5. UNCERTAINTY BUDGET FOR CALIBRATION OF WEIGHING INSTRUMENTS

5.1 Manufacturers of weighing instruments including electronic, electro-mechanical and mechanical balances have traditionally specified repeatability, eccentricity and linearity separately. In order for the calibration laboratory to claim compliance with specification they need to provide objective evidence based on measurements that the sum of the measurement error and uncertainty does not exceed the stated accuracy.

5.2 The need to claim compliance is further complicated as specifications are not necessarily stated as outside limits, for example the repeatability is often specified at ±1 σ, and balance specifications are established under ideal conditions, whilst the balance submitted for calibration is located in a working environment in which the operating conditions may be within specification but not necessarily ideal.

5.3 Laboratories may adopt a simplified approach in practice in the estimation of the measurement uncertainty for the linearity measurement of weighing equipment having fewer than 30 000 divisions, whereby the uncertainty is estimated by taking into account the uncertainty of the weights, the eccentricity (corner load), repeatability, and/or the resolution of the weighing instrument (whichever is larger).

5.4 The standard uncertainty due to the eccentricity (Ue) is estimated as

$ue = [d1/d2 * D] * 1/\sqrt{3}$ (adapted from R111 / R 76)

Where:

d1 = the distance from the centre of the load receptor to the centre of the weights
d2 = the distance from the centre of the load receptor and one of the corners
D = the deviation between the maximum and minimum measured values

The ratio of d1/d2 will typically be either 2/3 or 2/4 (See SADCAS TR 16)

5.5 Laboratories shall supply (and have available) for evaluation all relevant data of the weighing instrument, such as specification sheets, used in the determination of the calibration and measurement capability (CMC), along with copies of the Uncertainty of Measurement estimation. All relevant factors shall be considered, and only after confirmation that the contribution is insignificant may they be omitted from the estimation. This process shall be documented.

5.6 Laboratories are required to consider as a minimum the following contributors when establishing the uncertainty of measurement of weighing instruments having in excess of 2 000 000 divisions (typically analytical, semi and micro balances):

- The uncertainty of the weights;
- The stability of the weights;
- Buoyancy of the weights;
- The repeatability of the weighing instrument;
- The resolution of the weighing instrument;
- Temperature coefficient of sensitivity;
- The eccentricity (corner load)

Where the repeatability of the balance is found to be zero, the repeatability stated by the manufacturer shall be assumed.

6. REFERENCES

- SADCAS TR 12 : Estimation of the uncertainty of measurements by calibration laboratories and specification of calibration and measurement capability on schedules of accreditation
- SADCAS TR 16 : Criteria for Laboratory Accreditation in Mass Metrology
- JCGM 100: 2008 : Guide to the Expression of Uncertainty in Measurement (GUM)
- OIML R 111-1 : International Recommendation R 111-1 : 2004 (E) Weights of Classes E1, E2, F1, F2, M1, M2, M3
- GUM : Guide to Expression of Uncertainty in Measurements (GUM)

APPENDIX A: EXAMPLE OF UNCERTAINTY BUDGET FOR WEIGHTS

UNCERTAINTY BUDGET								
Description:			**Type:**			**Metrologist:**		
Symbol	**Description**	**Type**	**Expected Value**	**Probability distribution**	**Divisor**	**Sensitivity coeficient**	**Uncertainty contribution**	**Degree of freedom**
U_w	Repeatabili	A		Normal	1	1		
$U(m_{cr})$	Calib of std	B		Rectangular	2	1		
U_b	Buoyancy	B		Rectangular				
U_{ba}	Resolution	B		Rectangular	1.732051	1		
				Combined standard uncertainty				
				Expanded Uncertainty				
				Degree of freedon V_{eef}				
				Reliability				

APPENDIX B: EXAMPLE OF UNCERTAINTY BUDGET FOR WEIGHING INSTRUMENTS

Symbol	Uncertainty Source	Uncertainty Estimate [g]	Probability Distribution	Divisor	Sensitivity Coefficient	Uncertainty Contribution [g]		Degrees of freedom
Ws	Weights		Normal	2	1			
Ds	Drift		Rectangular	1.732050808	1			
d	Resolution		Rectangular	1.732050808	1			
E	Eccentricity		Rectangular	1.732050808	1		If applicable	
CS	Cold start drift		Rectangular	1.732050808	1		If applicable	
Wr	Repeatability		Normal	1	1		n-1 =	
u(Wx)			Normal		Combined std. uncertainty		ν_{eff}	
U					Expanded uncertainty		Coverage factor	2.00

1)	**W_s :** Get the uncertainty of the standard from its calibration certificate
2)	**δDs:** The drift uncertainty may be estimated from its history: difference between last and second last / (months between last and second last calibrations) x (months since last calibration).
	If more than one mass piece is used to make up for the total reference weight, then an algebraic sum is considered since weights are correlated.
	It the reference weights have no history, drift is estimated to be equal to the uncertainty reported on the calibration certificate.
3)	**δId:** The resolution uncertainty is $\delta I_d/2$. Take the resolution specified for the balance (d) and divide by 2. (i.e. half-width of a rectangular distribution).
4)	**δE :** Eccentricity, Mostly included already in repeatability uncertainty OR use two-thirds of difference between minimum and maximum readings obtained during eccentricity test, and treat as half-width of a rectangular distribution.
5)	**δCs:** Cold start drift. Use an estimate of zero of balance is normally switched on all the time. Else use the difference between the largest and smallest readings and treat it as having as a rectangular distribution.
6)	**δWr :** Repeatability, take ESDM of the repeated readings. If measurements give zero standard deviation (for low resolution balances) use manufacturer's repeatability

7)	Divide uncertainty values by divisors												
8)	**Combined standard uncertainty:** Sum the squares of the resulting uncertainty contributions and take the square root.												
9)	**ν_{eff} :** Effective degrees of freedom, take 4th power of resulting combined uncertainty and divide by sum of the (4th powers of the uncertainty contributions with non-infinite degrees of freedom divided by their degrees of freedom).												
10)	**Confidence level:** Look up the corresponding coverage factor k in the attached Student's- t table and multiply the combined std. uncertainty with it to get the expanded uncertainty giving a 95.45% confidence level												

APPENDIX C - AMENDMENT RECORD

Revision Status	**Change**			**Approved by**	**Effective Date**
	Page	**Clause/ Subclause**	**Description of Change**		
Issue 1	-	-	-	SADCAS CEO	2018-04-01